Introduction to Value-at-Risk (VaR)

Executive Summary

- Introduction and history of VaR
- Three commonly used methods of quantifying VaR
- Benefits and limitations
- Case study on Citigroup

Contributors:

Ow-Yang Zhi Yan
Z090086@e.ntu.edu.sg

Zhou Xinzi
ZHUO0184@e.ntu.edu.sg

Chia Zhiwei Isaiah
ZICHLIA1@e.ntu.edu.sg

Christoffer Alejandro Neil Ng
Meng Hui
CANG1@e.ntu.edu.sg

1.0 Introduction

For many that have invested or are considering investing in risky assets, it is often common to be concerned about the maximum amount an investor can possibly lose on a particular investment decision. Value at Risk (VaR) aims to provide a reasonable bounds answer. Due to the widespread use of VaR as a tool for risk assessment, especially in financial service firms, this report aims to briefly introduce to the reader the background, basic methods, benefits and limitations of the VaR tool. A case study on Citigroup will also be examined.

By basic definition, VaR measures the potential loss in value of an asset over a defined period for a given confidence interval. Any meaningful description of Value at Risk should comprise of three components:

1. Time period,
2. Confidence level, and
3. VaR Amount (Loss Amount)

Hence, a typical VaR statement might look like this: “VaR on [Asset] is $100 million within a one-week interval, 95% confidence level”

While VaR can be used by any entity to measure its risk exposure, it is widely used by commercial banks, investment banks and most financial institutions to capture the potential loss in value of their assets or portfolios from adverse market conditions over a specific period of time. This potential loss
will then be compared against their available capital and cash reserves to ensure balance sheet solvency.

VaR tries to capture the downside risk and potential losses. It is used by financial firms to prevent an insolvency crisis, where a low-probability catastrophic occurrence creates a loss that wipes out the firm’s entire capital base. It is to note that VaR can be defined more broadly or narrowly in specific contexts, measuring risks in the form of interest rate changes, equity market volatility, and economic growth.

1.1 History and Development of VaR

The impetus for the use of VaR measures came from the crisis that beset financial firms over time and the regulatory responses to this crisis. In the aftermath of the Great Depression, Securities Exchange Commission (SEC) required banks to keep borrowings below a specific equity capital. In the decades thereafter, banks have consistently devised risk measures and control devices to ensure that they meet these capital requirements.

With increased risk associated to derivatives markets and floating exchange rates in the early 1970s, capital requirements were further refined. In 1980, SEC tied the capital requirements of financial firms to the losses that would incur with 95% confidence over a thirty-day interval in different security classes. In addition to stricter regulatory reporting demand, trading portfolios of investment banks and commercial banks were becoming larger and more volatile. A more sophisticated and timely set of risk control measures were thus needed.

By 1990s, many financial firms had already developed rudimentary measures of Value at Risk, with wide variations on how it was measured. However, it was only in the aftermath of numerous disastrous losses associated with the use of leverage and derivatives between 1993 and 1995, coupled with the failure of Barings, that firms were ready for a more comprehensive risk measures.

In 1995, J.P. Morgan provided public access to data on the variances and co-variances across various security and asset classes it had used internally to manage risk. This allows software makers to develop software, based on the matrices made public and available to measure risk. The measure was readily received by the commercial and investment banking, while regulatory authorities welcomed this intuitive appeal. In the last decade, VaR has being widely used by financial firms and is finding acceptance in non-financial firms.

2.0 Methodology

Next, we discuss the three popular computational methods used in calculating Value-at-Risk:

2.1 Historical Simulation (Empirical Distribution Function)

In the historical simulation method, we plot the empirical distribution of the metric (eg asset price, asset return) that we are interested in, according to historical data. In other words, we do not assume any theoretical distribution in this method and are therefore not interested in the mean, standard deviation or any distribution parameter of the historical data. Instead we are only
concerned about the actual frequencies of different occurrences in the historical distribution (within a defined period).

After plotting the empirical distribution, we assume that history will repeat itself and that all occurrences are independent and identically-distributed (iid). We then read off the results on the left-tail of the curve to determine the worst outcome within our desired confidence level.

![Distribution of Daily Returns](image)

*Figure 1: 1387 daily returns of stock QQQ. Source: (Investopedia)*

In the graph above, results of daily returns on the stock QQQ were collected on 1387 occasions and plotted with the percentage return on the horizontal axis (worst returns on the left and best returns on the right) against corresponding frequencies on the vertical axis. According to the graph, the worst 5% of daily returns ranged from -4% to -8%. Therefore, in this situation, we can expect that our daily return will exceed -4% with 95% confidence.

Other variations of the historical simulation method include the weighted historical simulation and the filtered historical simulation.

### 2.2 Parametric (Variance-covariance) method

Under the parametric method, we make the strict assumption that our test subject follows a familiar distribution, usually the Normal distribution. Based on historical data, we calculate the most likely parameters (e.g., mean and standard deviation) of the distribution and make necessary projections with the use of these parameters.
As seen from the graph above, based on historical data (histogram), the likely mean and standard deviation parameters are calculated, leading to the plotted theoretical normal distribution. The red area represents the Value at Risk at 95% confidence.

The benefit of the parametric method over the historical simulation is that we know more properties of the theoretical normal distribution than we know of the true distribution in historical simulation. This grants us greater perceived knowledge of the risk distribution.

This method however, is not recommended for long horizons, portfolios with many options, or assets with skewed distributions.

Example of Parametric calculation:

We express VaR = Market Price * Volatility [which is usually derived by: confidence interval*standard deviation]

We have two assets within a portfolio, A and B
- **Price of A** follows a normal distribution, with standard deviation 1%
- **Price of B** follows a normal distribution, with standard deviation 1.5%
- **Correlation between A and B is 0.6**

Calculate the VaR at 95% confidence interval:

<table>
<thead>
<tr>
<th>Portfolio Value</th>
<th>Volatility * 95% Confidence Level Scaling Factor</th>
<th>VaR</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1,000,000</td>
<td>1%*1.65</td>
</tr>
<tr>
<td>B</td>
<td>$1,000,000</td>
<td>1.5%*1.65</td>
</tr>
</tbody>
</table>
We use the formula:
\[ VaR = \sqrt{(VaR_1^2 + VaR_2^2 + 2 \times Correlation_{1,2} \times VaR_1 \times VaR_2)} \]
\[ = [16,500^2 + 24,750^2 + 2 \times 0.6 \times 24,750 \times 16,500]^{1/2} \]
\[ = 37079.138 \text{ (being the Value at Risk of the portfolio at 95% confidence level)} \]

Numerically, you might have also realized that the smaller the correlation between A and B, the smaller the VaR for the portfolio of A and B as well.

2.3 Monte Carlo Simulation

The Monte Carlo method is the most complex and time-consuming of the three popular methods for calculating VaR. In essence, the Monte Carlo simulation method involves the full re-pricing of an asset based on algorithms that factor in current price, time, a number generated from a generator, and other factors.

Under one single Monte Carlo simulation, we simulate one single scenario of the movements of the asset price over a desired time period: the computer program generates a single price value and/or a single return value of the asset corresponding to each time frame (e.g., at time T, price = 1; at time T+1, price = 1.2; at time T+2, price = 1.4 etc). This is called a “path”:

The standard stock price model used to simulate the path of a stock price from the ith day:

\[ R_i = \frac{(S_{i+1} - S_i)}{S_i} = \mu \delta t + \sigma \phi \delta t^{1/2} \quad (1) \]

Where

- \( R_i \) is the return of the stock on the ith day
- \( S_i \) is the stock price on the ith day
- \( S_{i+1} \) is the stock price on the i+1th day
- \( \mu \) is the sample mean of the stock price return
- \( \delta t \) is the timestep
- \( \sigma \) is the sample volatility (standard deviation) of the stock price return
- \( \phi \) is a random number generated from a normal distribution (or any other distribution)
We then repeat the simulation over thousands or millions of times to generate thousands or millions of “paths”. Hence, we would have simulated numerous different asset prices at the end of the desired time period. Then we rank the simulated prices accordingly from best to worst and read off the worst result within our confidence level to determine the VaR within the desired time period.

2.4 Comparing VaR over different time horizons

Due to time variable, users need to know how to convert one value at risk (VAR) of one time period into the equivalent VAR for a different time period. For example, if the “worst daily return” of a stock is -2% at a 95% confidence interval, what would be the “worst monthly return” of the stock at a 95% confidence interval?

We can easily calculate this by multiplying the daily return with the square root of the time period. In the above example, the “worst monthly return”, assuming that there are 22 trading days within a month, would be 9.38% \([= (-2\%) \times (22)^{1/2}]\)

This is due to the classic mathematical formula: \(\sigma(aX) = \sqrt{a} \cdot \sigma(X)\)

Note that this conversion is only applicable for the Parametric method (Section 2.2).

3.0 Benefits and Limitations

The VaR model has been touted as one of the greatest innovation in providing a summary measure of risk. Take for instance a financial institution looking to hedge exposed positions in the options derivative market. Analysts will compute the “Greeks” such as delta-gamma-vega, resulting in thousands of different market variables which are difficult to interpret in aggregate. VaR attempts to provide a single number which summarizes all the risk in a portfolio of financial assets. We will delineate the benefits and limitations of the VaR model in the following section.

3.1 Benefits of VaR

VaR serves as a comparable benchmark measure of risk across different divisions spanning different markets in the world. Under this usage scenario, VaR is utilized to track the changes of risk over a
time for use in a time-series analysis. In addition, a cross-sectional analysis can be performed by comparing the different divisions in a certain time period to ensure compliance with the pre-determined risk tolerance of the institution.

The VaR measure could be employed to maintain an equity cushion for a firm, effectively serving as a measure of equity capital for a financial institution. A financial institution will determine an adequate risk buffer based upon its risk aversion, and consequentially adjust the input of the confidence level into the VaR model. This will allow the institution to not only ensure regulatory compliance, but also effectively control its leverage adequately to avoid bankruptcy should the market move in an unfavorable direction.

3.2 Limitations of VaR

As often as VaR is employed as first line of defense against market risk, practitioners should be reminded of one of its limitations: the model does not provide an absolute loss estimate. As discussed in the previous sections, the VaR benchmark provides a loss estimate at a pre-determined level of confidence, and does not account for black swan events nor for increased kurtosis during a crisis, which will increase the maximum absolute losses the institution could potentially incur.

VaR inherently assumes that the market exposure of the firm is fixed over a specified time horizon, thus failing to account for differing risk exposure over the period. Volatility and the VaR measure are often scaled by the square-root-of-time variable method to obtain a measure for the time horizon of interest. The scaling factor does not account for change in exposure an institution would have taken in response to changing market conditions.

Historical volatility data are used as inputs into the VaR model, which inherently assumes that historical data proves to be a good predictor of the future. As such, stability risk exists because historical data fails to account for structural surprises such as new regulatory requirements and increase in correlation coefficients across financial assets during a crisis. The failure to incorporate new information to predict future volatility will provide a grave underestimation of the risk.

Where historical data is not available as an input for the model, information risk does exist. For instance, in situations such as the pricing of risk for an initial public offering, in a private placement, or derivatives traded in the over-the-counter market, proxies are used in these scenarios, and the reliability and accuracy of the estimate is highly dependent on the supposed correlation between the proxy of choice and the financial instrument of interest.

4.0 Case Study

Having learnt the various aspects of value at risk, let us now look at how VaR played a role in the 2008 financial crisis.

In 2008, a typical investment portfolio of 60% stocks and 40% bonds lost about 20% of its value. Standard portfolio-construction tools assume that such an event will only happen once every 111 years. Using a fat-tailed distribution, however, such an occurrence was predicted to possibly happen once every 40 years. This number obtained might be due to the fact that number crunchers who have a smaller supply of historical observations to construct models tended to focus on rare events.
The “Value-at-Risk” model is widely used due to the ease of understanding compared to the other available risk measures. In fact, it is so easy to understand that some bank managers may have gone one step further from using VaR as a risk measure to relying on it for risk management.

However, the Extreme Value Theory (EVT), a branch of probability statistics designed to deal with fat tails and extreme events, points to evidence that the commonly used VaR is overly-optimistic. Using data for Citigroup at the end of 2007 with a normal distribution tail for comparison, it is shown that using EVT (which is a fat tail extreme) an expected shortfall would be -9.7% instead of an overly optimistic -5% as indicated by the normal 99% VaR.

The EVT model (solid lines) says the 1-day 99% Expected Shortfall is -9.7%.

From a normal model (dashed lines) it is only -5%.

The normal model is hopelessly over-optimistic.

The following data on Citigroup, presented in Figure 4, was computed from the worst loss observed in the daily returns on a 250 day rolling data window from January 2007 to April 2009 at market close on the last trading day of each month. The expected shortfall conditional on exceeding this loss was compared with the average breach of the previous loss, over the subsequent months.

The results produced by Omega Analysis indicated that as the market was heading into a decline during July 2007, the error between the 99% VaR and the 99% expected shortfall increased exponentially, indicating the failure of the VaR model during a crisis. There was a severe inability to reflect actual “value at risk”.

![Figure 3: Fit look for Citigroup at the end of 2007, with a Normal distribution tail for comparison](image-url)
Many managers who heavily relied on the VaR model had heavily leveraged their portfolios. Depending on the degree of leverage, losses could be painfully magnified. The interaction of extreme leverage and the rapidly falling value of whatever was held in the portfolio led to the wipe-out of vulnerable firms, and eventually mergers, bailouts, and loss of confidence in the financial system. The implication is that the misuse of a risk measure, VaR, led to a degree of unwarranted comfort with the leveraged position of the firm.

The findings from this case were not specific to Citigroup or the Dow Jones Index alone; similar analysis revealed very similar situations for other entities such as Lehman Brothers, Halifax Bank of Scotland, Royal Bank of Scotland, BNP Paribas, ING. Equity indices worldwide and other asset classes such as hedge fund indices also showed similar characteristics. The use of other EVT methods also produced similar results.

As many investors were clobbered by the financial crisis and many investment firms lost significant portions of their assets under management, enhanced risk management has become a current priority for most in the industry. VaR, however, will continue to be used as a way to communicate risk.

5.0 Conclusion

VaR is not necessarily less effective than other risk measures used, since standard deviation is one of the most commonly used risk measures in the industry. VaR improves on standard deviation in that standard deviation is not an intuitive concept to most investors. VaR is stated in terms of the dollar value of the portfolio, so it is a more intuitive estimate of risk. However, exclusive reliance on VaR without due consideration of the tail risk of extreme losses had clearly led to painful losses. The lessons we should take away from the financial crisis include the need to watch out for over-reliance on VaR or any one tool alone. We need to prepare for downside events beyond the “expected downside” quantified by VaR, even if these events are of a small probability. A proper understanding of VaR should lead to productive discussions of the risk of extreme losses, the investor’s appetite for such risk, and strategies for managing the risk-return trade-offs.
References


Investopedia (n.d). An Introduction To Value at Risk (VAR), extracted from: http://www.investopedia.com/articles/04/092904.asp#axzz2Ka4jT4Ng


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