PH1103/PH1105 Optics, Vibrations and Waves
AY1415 Final Exam Suggested solutions

by some kind soul

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1 Vibration and Waves

(a) k = 14.1 m^{-1} ; $\omega = 166$ rad/s
$\lambda_4 = \frac{2\pi}{k} = \frac{2\pi}{14.1} = 0.446$ m
$\boxed{\lambda_1 = 4\lambda_4 = 1.78\,m}$
$f_4 = \frac{\omega}{2\pi} = \frac{166}{2\pi} = 26.42$ Hz
$\boxed{f_1 = \frac{4}{4} = 6.6\,Hz}$

(b) Fundamental mode. L = $\frac{\lambda}{2}$
Third Overtone, L = $\frac{\lambda}{2} \times 4 = \frac{4\times 0.446}{2} = 0.891$ m

(c) $v = f \times \lambda \implies v = 0.446 \times 26.42 = 11.78$ m/s

2 Doppler Effect

(a) The change in frequency of a wave (or other periodic event) for an observer moving relative to its source (Wikipedia)

(b) Formula for Doppler Effect as follow

$$f_L = \frac{v \pm v_L}{v \pm v_S} f_S \tag{2.1}$$

where the sign depend on the direction of each subject. Positive direction defined as direction from listener to source

i) Case 1: BEFORE police car passed the passenger vehicle, source approaching from behind

$$f_L = \frac{v - v_L}{v - v_S} f_S = \frac{340 - 10}{340 - 30} f_S = \frac{330}{310} f_S = 468.4\,Hz$$

ii) Case 2: AFTER police car passed the passenger vehicle, source moving away from listener

$$f_L = \frac{v + v_L}{v + v_S} f_S = \frac{340 + 10}{340 + 30} f_S = \frac{350}{370} f_S = 416.2\,Hz$$

3 Geometrical Optics

(a) Angular Magnificiation,

$$M = \frac{\theta'}{\theta} = \frac{-y'/f_2}{-y'/f_1} = -\frac{f_1}{f_2} = -\frac{44}{4} = -11$$

$f = 4.0$ cm should be used as the eyepiece

(b) Planoconvex lens $\implies R_1 = \infty; f = 18.7$ cm $; n = 1.458$
Lens-Maker Equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{3.1}$$

$$\frac{1}{f} = (n-1)\left(-\frac{1}{R_2}\right) \quad (R_1 = \infty, \frac{1}{R_1} = 0)$$

$$\implies R_2 = -(n-1)f = 0.458 \times 18.7 = \mathbf{8.565 \quad cm} \tag{3.2}$$

(c) Formula for Quartz Rod

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \tag{3.3}$$

$$\frac{1}{20} + \frac{1.458}{8.15} = \frac{0.458}{R}$$

$$\implies R = \frac{0.458}{\frac{1}{20} + \frac{1.458}{8.15}} = 2.0\,cm \tag{3.4}$$

With result 3.4, we can do the calculation when immersed in water as follow

$$\frac{1.33}{20} + \frac{1.458}{s'} = \frac{1.458 - 1.33}{2}$$

$$\implies s' = \frac{1.458}{-\frac{1.33}{20} + \frac{0.128}{2}} = \mathbf{\text{-}583 \quad cm}$$

(d)
- Start with Lens Maker Equation for Symmetric Double Convex Lens

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (n-1)\frac{2}{R}$$

$$\implies R = 2(n-1)f = 2 \times 0.52 \times 40 = 41.6\,cm \tag{3.5}$$

- Use result 3.5 as R in the following part. Work on 1st Refraction Surface (air-lens)

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \tag{3.6}$$

$$\implies \frac{1}{70} + \frac{1.52}{s'} = \frac{0.52}{41.6}$$

$$\implies s' = \frac{1.52}{\frac{0.52}{41.6} - \frac{1}{70}} = \mathbf{\text{-}851 \quad cm} \tag{3.7}$$

- With result 3.7, we can calculate for the 2nd refraction surface (lens-water) with $s_2 = 851$

cm

$$\frac{1.52}{851} + \frac{1.33}{s_2'} = \frac{-(1.52 - 1.33)}{-41.6} = \frac{(1.33 - 1.52)}{-41.6}$$
$$\implies s_3' = \frac{1.33}{\frac{0.19}{41.6} - \frac{1.52}{851}} = 478.2\,cm \tag{3.8}$$

- Because the mirror is only 90 cm away, mirror reflect the image back. $478.2 - 90 = 388.2$ cm to the left of the mirror but $388.2 - 90 = 298.2$ cm to the left of the lens
- Now, reverse apply for the water-lens interface

$$\frac{1.33}{-298.2} + \frac{1.52}{s_3'} = \frac{1.52 - 1.33}{41.6}$$
$$\implies s_3' = \frac{1.52}{\frac{0.19}{41.6} + \frac{1.33}{298.2}} = 168.3\,cm \tag{3.9}$$

- Apply result 3.9 to the last lens-air interface

$$\frac{1.52}{-168.3} + \frac{1}{s_4'} = \frac{1 - 1.52}{-41.6}$$
$$\implies s_4' = \frac{1}{\frac{0.52}{41.6} + \frac{1.52}{168.3}} = \mathbf{46.44 \quad cm} \tag{3.10}$$

- $M = m_1 m_2 m_3 m_4 = \left(\frac{\cancel{-851}}{70}\right)\left(\frac{478}{\cancel{851}}\right)\left(\frac{\cancel{168.3}}{-298}\right)\left(\frac{46.44}{\cancel{-168.3}}\right) = -1.06$
- The image is an **inverted real image**.

4 Interference and Diffraction

(a) Always half-cycle phase shift in the air-glass interface

(b) $m = 1; 2d\sin(\theta) = m\lambda \implies d = \frac{\lambda}{2\sin(\theta)} = \frac{0.165\,\text{nm}}{2\sin(23.5°)} = 2.07\,\text{Å}$

(c) $\sin(\theta) \approx \theta = 1.22 \times \frac{\lambda}{D} = 1.22 \times \frac{0.6328\,\mu\text{m}}{7\,\mu\text{m}} = 0.11$
$\theta = \frac{Radius}{distance} = \frac{d/2}{l} = \frac{d}{2l} \implies d = 2l \times \theta = 9.0\,\text{m} \times 0.11 = 0.99\,\text{m} \approx 1\,\text{m}.$ $\boxed{r \approx 0.5\,\text{m}}$

(d) Optical path $2t$. Half cycle phase shift. The bright fringes occur at $2t = (m + \frac{1}{2})\frac{\lambda}{n}$
When $t = h$, $2h = (m + \frac{1}{2})\frac{\lambda}{1.5} = 2l\theta$, since $\frac{h}{l} = \theta$ by small angle approximation.
Let $m = 20$, $l = 1\,\text{mm}$

$$(20 + \frac{1}{2}) \times \frac{550\,\text{nm}}{1.5} = 2 \times 10^6 \cdot \theta \implies \boxed{\theta = \frac{20.5 \times 550}{1.5 \times 2 \times 10^6} = 3.76 \times 10^{-3}\,\text{rad} = 0.215°}$$

5 Photoelectric Effect (PH1103 Only)

Formula for Photoelectric Effect as follow:

$$E = \frac{hc}{\lambda} \tag{5.1}$$

where h is Planck Constant, c is speed of light and λ is the wavelength of specific light.
Applying the formula above, we have

$$E_{threshold} = E_0 = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{272\,\text{nm}} = 7.3 \times 10^{-19}\,\text{J} = 4.56\,\text{eV}$$
$$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{206\,\text{nm}} = 9.6 \times 10^{-19}\,\text{J} = 6.02\,\text{eV}$$

Kinetic Energy of ejected photon are the difference of energy from incoming light with threshold energy i.e

$$K = E - E_0$$

$$K = 6.02\,\text{eV} - 4.56\,\text{eV} = 1.46\,\text{eV}$$

END OF SOLUTION