PH1104/PH114S - MECHANICS

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MULTIPLE CHOICE ANSWERS

1. (E) the first four options are clearly wrong since v_x needs to change its sign at a moment during the motion and there's no way v_x could switch at an instant from positive to negative (or vice versa). So we are left with the last option. Or you can think it this way; the glider accelerates with constant acceleration of $g \sin \phi$ towards downhill. Therefore, the curve of $v_x - t$ should have a constant gradient.

2. (C) as long as you remember cardinal directions and know how to apply Phytagorean theorem, it's easy.

3. (D) some amount of U_{grav} will be converted into U_{el}.

4. (A) Apply Newton's law of motion.

$$\sum F_x = ma_s \quad \Longrightarrow \quad N = m\omega^2 R$$

and

$$\sum F_y = 0 \quad \Longrightarrow \quad f_{s_{\max}} - mg = 0 \quad \Longrightarrow \quad \mu_s N - mg = 0 \quad \Longrightarrow \quad \omega_{\min} = \sqrt{\frac{g}{\mu_s R}}$$

As we can see, the larger ω results in larger N. Note that when $\omega > \omega_{\min}$, the person starts to slide up inside the cylinder since f_s has exceeded mg. So by the definition of static friction, in $\omega > \omega_{\min}$ case, the static friction is just the same as maximum static friction which does not play role anymore. Therefore, they both share the same value as in $\omega = \omega_{\min}$ case.

5. (A) the centre of mass of the system is proportionally closer to the larger mass.

6. (B) the star's moment inertia decreases as its radius decreases since $I \sim mr^2$. Recall that the rotational kinetic energy can be written as $T = L^2/2I$ or alternatively $T = L\omega/2$. By the first argument, it is obvious that T increases as I decreases. The other way to solve the problem is by knowing the fact that the star's angular momentum ($L = I\omega$) has to be conserved, thus decreasing I will lead the star to increase its ω. Therefore, T will increase as ω increases.

7. (C) Just in case you're not aware, Pluto moves in elliptical orbit instead of circular orbit. Recall Kepler's law of orbits. Thus, the force of gravity in elliptical orbit does work on Pluto since there is a component of the force in the direction Pluto moves.

$$W = \int_C \overbrace{\mathbf{F} \cdot d\mathbf{s}}^{\neq 0} \quad \Longrightarrow \quad W \neq 0$$

Recall Kepler's second law, since Pluto moves much faster at perihelion than it does at aphelion then the work done $(W = \Delta T = -\Delta U)$ will be negative. But note that for **any** type of path an object moves in gravitational field, the work done in going around that path is **zero**. It follows from

$$W = \oint \mathbf{F} d\mathbf{s} = 0$$

8. (B)The pulley will rotate anti-clockwise since $m_1 > m_2$. Analyse the torque on pulley's centre,

$$\sum_i \tau_i = I\alpha$$
$$T_1 R - T_2 R = I\alpha$$
$$(m_1 - m_2)\, gR = I \frac{(\omega - \omega_0)}{t}$$

By rearranging the expression above, we have

$$t = \frac{I\omega}{(m_1 - m_2)\, gR} \approx 2\,\text{seconds}$$

9. (A) the hint in the question is the key to solve this problem. From the hint given, it's easy to see that $I/M < I'/M'$ thus $IM'/I'M < 1$. By comparing the rotational kinetic energy, we have

$$\frac{K_R}{K'_R} = \frac{\frac{1}{2} I\omega^2}{\frac{1}{2} I'\omega'^2} = \frac{Iv^2}{I'v'^2} = \frac{2IK_T/M}{2I'K'_T/M'}$$

By the magical algebra, we have

$$\frac{K'_T}{K'_R} \cdot \frac{K_R}{K_T} = \frac{I/M}{I'/M'} \quad \Longrightarrow \quad \boxed{\frac{K'_T}{K'_R} < \frac{K_T}{K_R}}$$

10. (B) the sum of torques (on O) should be zero for the bar to be in equlibrium.

$$\sum_i \tau_i = 0 \quad \Longrightarrow \quad Fl - mg\sin\theta \cdot \frac{1}{2} l = 0 \quad \Longrightarrow \quad \boxed{F = \frac{1}{2} mg \sin\theta}$$

SOLUTION #11

(a) Let's gather all quantities given in the question.

$\implies H = 400\,\mathrm{m},\, v_0 = 50\,\mathrm{m/s},\, \phi = 30°,\, \phi = 30°,\, D = 100\,\mathrm{m},\, L = 200\,\mathrm{m}$

So basically we are just required to find the horizontal distance at the moment the boulder touches the ground.

$$y = y_0 + v_{0_y} t - \frac{1}{2} g t^2 \quad \implies \quad 0 = H - v_0 \sin\phi\, t - \frac{1}{2} g t^2$$

Solving the quadratic equation, we have

$$t = \frac{-v_0 \sin\phi + \sqrt{v_0^2 \sin^2\phi + 2gH}}{g} \approx 6.8\,\text{seconds}$$

Thus the maximum horizontal distance χ reached by the boulder is

$$\chi = v_{0_x} t = v_0 \cos\phi \left[\frac{-v_0 \sin\phi + \sqrt{v_0^2 \sin^2\phi + 2gH}}{g} \right] \approx 296\,\text{meter}$$

It turns out that she was right.

At the moment it touches the ground, its velocity components are

$$\begin{aligned} v_x &= v_{0_x} & \implies \quad & v_x = 25\sqrt{3}\,\mathrm{m/s} \\ v_y &= v_{0_y} - gt & \implies \quad & v_y \approx -42\,\mathrm{m/s} \end{aligned}$$

Then the boulder's speed at the moment it hits the pond is $\sqrt{\left(25\sqrt{3}\right)^2 + (-42)^2} \approx 60\,\mathrm{m/s}$.

(b) On the smaller block:

$$\sum F_x = m_2 a_2 \quad \implies \quad f_2 = m_2 a_2 \quad \implies \quad a_2 = \mu_s g$$

For the smaller block not to slide on the bigger block, they must have the same acceleration.

$$\sum F_x = m_1 a_1 \quad \implies \quad F - f_2 = m_1 a_1 \quad \implies \quad F_{\max} = (m_1 + m_2)\, g\mu_s = 17.64\,\mathrm{N}$$

If F exceeds the value above, the smaller block will move with acceleration $a_2 = \mu_k g = 1.96\,\mathrm{m/s^2}$. While the bigger block will move with acceleration

$$a_1 = \frac{2F - m_2 a_2}{m_1} = 7.84\,\mathrm{m/s^2}$$

If F does not exceed the maximum value, they will move with the same acceleration.

$$\frac{1}{2}F_{\max} - \overbrace{f_2}^{m_2 a} = m_1 a \quad \Longrightarrow \quad a = \frac{F_{\max}}{2\left(m_1 + m_2\right)} \quad \Longrightarrow \quad f_2 = \frac{m_2 F_{\max}}{2\left(m_1 + m_2\right)} \approx 2.94\,\mathrm{N}$$

So the friction force acting on the smaller block will be 2.94 N while the friction force acting on the bigger block is just the opposite sign of the one acting on the smaller block.

(c) By conservation of energy

$$mgl\left(1 - \cos\phi\right) = \frac{1}{2}mv^2 \quad \Longrightarrow \quad v = \sqrt{2gl\left(1 - \cos\phi\right)} \approx 12.5\,\mathrm{m/s^2}$$

By conservation of momentum

$$\begin{aligned} m_p v_p &= m_b v_b + m_p v_p' \\ m_p v &= \left(m_b - m_p\right)\left|v'\right| \\ m_b &= \frac{m_p v}{\sqrt{2gh'}} + m_p \end{aligned}$$

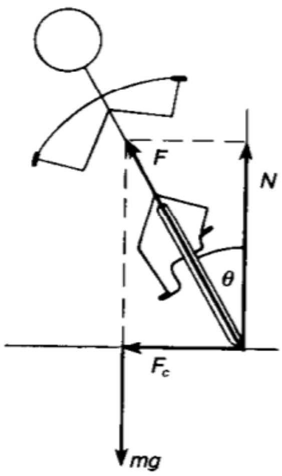
By working with the magical algebra, we arrived with

$$\boxed{m_b = m_p\left(1 + \sqrt{\frac{1 - \cos\phi}{1 - \cos\varphi}}\right)}$$

Putting all the values, we obtain $m_b \approx 4\,\mathrm{kg}$.

SOLUTION #12

(a)



For the motorcycle not to fall, the torque about the centre of gravity must be zero, which means that the vector force F exerted by the ground must have a line of action passing through the center of gravity. Thus

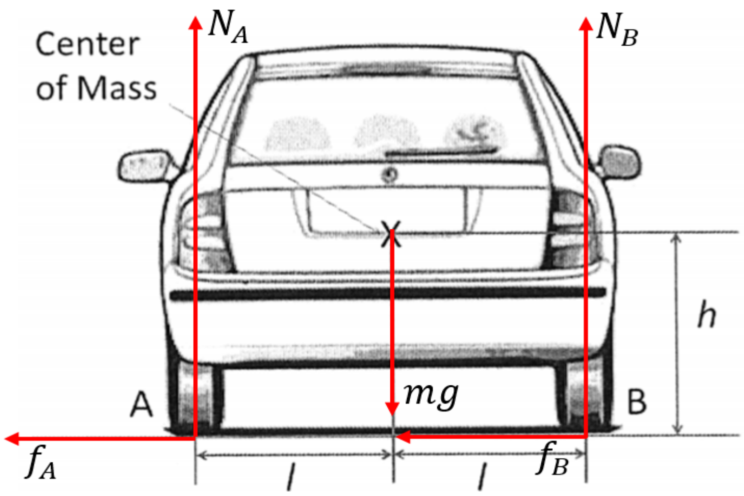
$$\tan\theta = \frac{F_c}{N} = \frac{mv^2/R}{mg} = \frac{v^2}{gR} \implies \boxed{\theta = \tan^{-1}\left(\frac{v^2}{gR}\right)}$$

If we take μ_s account, we will have

$$\tan\theta_m = \frac{F_c + f_s}{N} = \frac{v^2}{gR} + \mu_s \implies \boxed{\theta_m = \tan^{-1}\left(\frac{v^2}{gR} + \mu_s\right)}$$

which is greater than the previous case. This makes sense since f_s tends to prevent the motorcycle to slip i.e. it increases the lean angle at which motorcycle could reach to execute the turn perfectly.

(b)



Analyse the forces acting on the car.

$$\sum F_y = \overbrace{ma_y}^{0}$$

$$N_a + N_b = mg \tag{13.1}$$

and

$$f_a + f_b = m\omega^2 R + m\omega^2 (R + 2l) = 2m\omega^2 (R + l) \tag{13.2}$$

Analyse the total torque on the car's centre of mass.

$$\sum \tau_{\text{com}} = 0 \implies N_A l + f_A h + f_b h - N_B l = 0 \implies N_B - N_A = 2m\omega^2 h \frac{(R+l)}{l} \tag{13.3}$$

Eliminating (13.3) and (13.1) gives

$$\Longrightarrow N_A = \frac{1}{2}mg - m\omega^2 h\left(1 + \frac{R}{l}\right)$$
$$\Longrightarrow N_B = \frac{1}{2}mg + m\omega^2 h\left(1 + \frac{R}{l}\right)$$

As we can see, the normal force that will vanish is N_A with

$$\boxed{\omega_c = \sqrt{\frac{gl}{2h\,(l+R)}}}$$

and of course wheel B will lost touch with the ground. In the case $\omega > \omega_c$, the torque on the centre of mass should be zero for the car to not roll over.

$$\sum \tau = 0$$
$$N_A \sin\theta \cdot \sqrt{l^2 + h^2} + f_A \cos\theta \cdot \sqrt{l^2 + h^2} = 0$$

Note that for $\omega > \omega_c$, N_A will be negative and the outer wheels can't oppose the moment of centrifugal force anymore. Therefore, a car is more likely to roll over around a turn.

SOLUTION #13

(a) The angular velocity of the Earth's rotation about its axis is $\omega = \frac{2\pi}{T} = \frac{2\pi}{8.6\times10^4\,\text{s}} \approx 7.3 \times 10^{-5}\,\text{rad/s}$.
By Kepler's third law and considering satelite's mass is way much smaller than that of Earth's, we have

$$\frac{T^2}{(R+H)^3} \approx \frac{4\pi}{GM_\text{E}} \quad \Longrightarrow \quad H \approx 3.6 \times 10^7\,\text{meter}$$

The change in gravitational potential energy is

$$\Delta U = U_2 - U_1 = -\frac{GM_E m}{R+H} + \frac{GM_E m}{R} = mgh\frac{R}{R+H} \approx 2650\,\text{megajoules}$$

The change in kinetic energy is just the opposite sign of the change in gravitational energy, that is $-2650\,\text{megajoules}$.

(b) Let's denote mass of the Sun as $M_\odot$, the Earth's as M_E, and the sattelite's as m. Now analyse the Sun.

$$F_g = F_c$$
$$\frac{GM_\odot M_\text{E}}{R^2} = M_\odot\,\omega^2\,(R - R_\text{com})$$
$$M_\text{E}\,\omega^2 R_\text{com} = M_\odot\,\omega^2\,(R - R_\text{com})$$

We obtain

$$\boxed{R_{\text{com}} = \frac{M_\odot}{M_\odot + M_{\text{E}}} R}$$

Now analyse the satellite.

$$F_g = F_c$$
$$\frac{GM_\odot m}{(R - r_1)^2} - \frac{GM_{\text{E}} m}{r_1^2} = m\omega^2 (R_{\text{com}} - r_1)$$

From the previous result, we know that $R - R_{\text{com}} = \frac{M_E}{M_\odot + M_{\text{E}}} R$ therefore $\omega^2 = \frac{G(M_\odot + M_E)}{R^3} \approx \frac{GM_\odot}{R^3}$. By considering $R \gg r_1$ and $M_\odot \gg M_{\text{E}} \gg m$, the above expression becomes

$$\frac{GM_\odot}{R^2}\left(1 + \frac{2r_1}{R}\right) - \frac{GM_{\text{E}}}{r_1^2} = \frac{GM_\odot}{R^3}\left[\left(1 - \frac{M_{\text{E}}}{M_\odot}\right) R - r_1\right]$$
$$\frac{3M_\odot r_1}{R^3} - \frac{M_{\text{E}}}{r_1^2} = -\frac{M_{\text{E}}}{R^2}$$

By dividing both sides by M_{E}, we have

$$\frac{3M_\odot r_1}{M_{\text{E}} R^3} - \frac{1}{r_1^2} \approx 0 \implies \boxed{r_1 \approx R\sqrt[3]{\frac{M_E}{3M_\odot}}} \implies r_1 \approx 1.5 \times 10^9 \text{ meter}$$