Question 1

(a) Four properties of a laser:
   a. Coherent
   b. Monochromatic
   c. Collimated
   d. Directional

(b) “Temporal coherence” (longitudinal coherence) refers to the degree of coherence in the direction of wave propagation. Perfect longitudinal coherence for an optical wave implies that the planes of constant phase are uniformly spaced without interruption. Temporal coherence has important consequences for the spread of wavelengths in laser light.

   “Spatial coherence” (transverse coherence) refers to the degree of coherence along a wave front (perpendicular to the direction of wave propagation). Perfect spatial coherence for an optical wave means that the wavefronts are continuous, without interruptions in phase. Spatial coherence has important consequences for the directionality of laser beams.

(c) Two practical applications of large temporal coherence:
   a. Holography
   b. Fabry-Perot Etalon

(d) Amplification and lasing can be more easily achieved with a four-level system than in a three-level system because not much pump energy must be wasted in removing atoms from the ground state. For the three-level system, the lower laser level is the ground state, whereas for the four-level system the lower laser level is an excited state of the system, and this lower laser level decays quickly back to the ground state. Achieving population inversion ($N_2 > N_1$) in the three-level system requires that at least half the atoms be pumped out of the ground state, which requires a good deal of pump energy. In contrast, the four-level system can achieve population inversion with only a small number of atoms raised out of the ground state.
“Spectral hole burning” is a phenomenon which generates a ‘hole’ in saturation of inhomogeneous atomic lineshape. In this case, the light of certain frequency will only interact strongly with those atoms that have center frequencies within a homogeneous linewidth of that frequency. The gain from these “spectrally nearby” atoms will be saturated, whereas the gain from atoms in other parts of the lineshape spectrum will remain at the unsaturated value, resulting in a spectral ‘hole’ in the gain spectrum.

“Spatial hole burning” is a phenomenon which causes multimode lasing in the case of homogenously broadened atomic lineshape. It is a result from the fact that atoms in different spatial locations saturate independently with different transverse modes or different longitudinal modes.

**Question 2**

(a) The mode number is given by

\[ m = \frac{\nu}{c} = \frac{2L}{\lambda} \]

For \( \lambda_1 = 588.995 \text{ nm} \) and \( \lambda_2 = 588.592 \text{ nm} \), we obtain

\[ m_1 = (2 \times 0.15 \times 10^6)/(588.995) = 509.3 \]
\[ m_2 = (2 \times 0.15 \times 10^6)/(588.592) = 509.7 \]

Thus, the mode number of the resonance must be \( m = 509 \) (an integer).

(b) Having \( \Delta \lambda = 589.592 - 588.995 = 0.597 \text{ nm} \)

\[ m = \frac{2L}{\lambda} \Rightarrow \Delta L = \frac{m \cdot \Delta \lambda}{2} = \frac{509 \times 0.597}{2} \]

\[ \Delta L = 152 \text{ nm} \]

(c) From (b):

\[ m = \frac{2L}{\lambda} \Rightarrow \Delta L = \frac{\Delta m \cdot \lambda}{2} = \frac{1 \times 588.995}{2} = 295 \text{ nm} \]

Having the free spectral range

\[ \Delta \nu_{FSR} = \frac{c}{2L} = 10^{12} \text{ Hz} \]

\[ \Delta \lambda_{FSR} = \frac{\lambda^2}{c} \Delta \nu_{FSR} = \frac{c}{2L} = 1.156 \text{ nm} \]

\[ \Delta L = \frac{m \Delta \lambda_{FSR}}{2} = 294 \text{ nm} \]
Thus, the mirror spacing must be changed by **294 nm** in order to see the same emission line of different order.

Since \( m = \frac{2L}{\lambda} \), if the \( L \) increases, the new mode number must be higher.

(d) The frequency resolution

\[
\Delta v_{1/2} \approx \frac{1}{2\pi} \left( 1 - R^2 \right) \frac{c}{2L}
\]

\[
\Delta v_{1/2} = \frac{1}{2\pi} \left( 1 - 0.99^2 \right) \frac{3 \times 10^8}{2 \times 0.15 \times 10^{-3}} = 3.17 \times 10^9 \text{ Hz}
\]

In term of wavelength,

\[
\Delta \lambda_{1/2} = \frac{\lambda^2}{c} \Delta v_{1/2} = 3.67 \times 10^{-12} \text{ m}
\]

(We use the smallest wavelength according the definition of resolution.)

\[
\Delta \lambda_{1/2} = 3.67 \times 10^{-12} \text{ m}
\]

**Question 3**

(a) Having the atomic lineshape function

\[
g(\omega) = \int_{-\infty}^{\infty} E_0 e^{i\omega t} e^{-i/2\tau} e^{-i\omega t} dt
\]

\[
g(\omega) = \int_{0}^{\infty} E_0 e^{-i[(\omega - \omega_0) + i/2\tau]} \frac{\omega}{(\omega - \omega_0)^2 + (1/2\tau)^2} d\omega
\]

\[
g(\omega) = \frac{1}{i(\omega - \omega_0) + 1/2\tau}
\]

\[
g(\omega) = \frac{1/2\tau}{(\omega - \omega_0)^2 + (1/2\tau)^2} + \frac{-i(\omega - \omega_0)}{(\omega - \omega_0)^2 + (1/2\tau)^2}
\]

Hence, this is the Lorentzian lineshape.

(b) From the expression above, we obtain

\[
\frac{\Delta \omega}{2} = \frac{1}{2\tau}
\]

\[
\Delta \omega = \frac{1}{\tau}
\]
Question 4

(a) From the figure, the absorption cross section at 1480nm is
\[ \sigma_{abs}(1480) \approx 1.3 \times 10^{-21} \text{cm}^2 \]
The absorption coefficient is then
\[ \alpha = N \sigma = \left( 8 \times 10^{18} \text{cm}^{-3} \right) \left( 1.3 \times 10^{-21} \text{cm}^2 \right) = 1.04 \times 10^2 \text{cm}^{-1} \]

(b) The fraction of pump light absorbed is
\[ \text{Fraction} = 1 - e^{-\alpha L} = 1 - e^{-0.104 \times 200} = 0.875 \]

(c) With \( N_2 = 5 \times 10^{18} \text{ cm}^{-3} \) and \( N_1 \ll N_2 \), the gain coefficient is
\[ \gamma \approx N_2 \sigma_{em}(1560) = \left( 8 \times 10^{18} \text{cm}^{-3} \right) \left( 1.7 \times 10^{-21} \text{cm}^2 \right) = 1.36 \times 10^2 \text{cm}^{-1} \]
where \( \sigma_{em}(1560) \) is estimated from the figure.

(d) The net gain in a length of 2 m (or 200 cm) is
\[ G = \frac{I}{I_0} = e^{\alpha L} = e^{0.0136 \times 200} = 15.2 \]

Question 5

(a) Slope efficiency
\[ \eta_s = \frac{\Delta P_{out}}{\Delta P_{in}} = \frac{0.45 \text{ W} - 0.20 \text{ W}}{3.5 \text{ W} - 2.5 \text{ W}} \]
\[ \eta_s = 0.25 \]

(b) Having the relation
\[ P_{out} = \eta_s (P_{in} - P_{th}) \quad (*) \]
For \( P_{in} = 2.5 \text{ W} \) and \( P_{out} = 0.20 \text{ W} \), we get
\[ 0.20 = 0.25(2.5 - P_{th}) \]
Hence, \( P_{th} = 1.70 \text{ W} \)

(c) Again, using the expression (*), we obtain:
\[ P_{out} = 0.25(3.0 - 1.70) \]
\[ P_{out} = 0.325 \text{ W or 325 mW} \]

(d) We have a relation:
\[ \eta_s \approx \frac{T}{\delta + \frac{h \nu}{\lambda}} = \frac{T}{\delta + \frac{hc}{\lambda P}} = \frac{T}{\delta + \frac{\lambda P}{\lambda}} \]
\[ \delta = \frac{T \lambda P}{\eta_s \lambda} - T = \frac{0.02 \times 810}{0.25 \times 1060} - 0.02 \]
\[ \delta = 0.041 \text{ or 4.1%} \]
(e) We have a relation:

\[ \frac{2nL}{c\tau_c} \approx \delta + T \]

\[ \tau_c = \frac{2nL}{c(\delta + T)} = \frac{2 \times 1.5 \times 0.08}{3.0 \times 10^8(0.041 + 0.02)} \]

\[ \tau_c = 1.31 \times 10^{-8} s \text{ or } 13.1 \text{ ns} \]